

The Measurement of Social Exclusion*

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December 2002

Abstract

This paper develops an axiomatic approach to the measurement of social exclusion. At the individual level, social exclusion is viewed in terms of the number of functionings in the society from which the person concerned is excluded. At the aggregate level we treat social exclusion as a function of individual exclusions. The class of subgroup decomposable social exclusion measures using a set of independent axioms is identified. We then look at the problem of ranking exclusion profiles by exclusion dominance principle under certain restrictions. Finally, policy applications of our measures using European Union and Italian data are also considered.

JEL Classification No.: D63.

Keywords: Social Exclusion, Axioms, Characterization, Dominance, Application, Policy.

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1 Introduction

The subject of this paper is the measurement of social exclusion. The broad questions that we try to address in this paper are: (i) When do we say that an individual is socially excluded; (ii) What is the level of social exclusion in a country? (iii) Can we say that social exclusion in country A is less than that in country B? (iv) Given the level of social exclusion in a society, which subgroups of the population, partitioned according to ethnic, geographic, or any other socioeconomic characteristic, contribute more to aggregate social exclusion? (v) When can we say that one society dominates another with respect to social exclusion and what are the consequences of such a dominance relationship?

Broadly speaking, a person is said to be socially excluded if she/he is unable to ‘participate in the basic economic and social activities of the society in which she/he lives’. In the European Commission’s Programme specification for ‘targeted socioeconomic research’, social exclusion is described as ‘disintegration and fragmentation of social relation and hence a loss of social cohesion. For individual in particular groups, social exclusion represents a progressive process of marginalization leading to economic deprivation and various forms of social and cultural disadvantage’.

As Atkinson (1998) said, social exclusion is not just a consequence of unemployment. It is true that an unemployed person may not have income to maintain a subsistence standard of living and hence becomes socially excluded. But many employed persons may not be satisfied with their work or main activity. Expansion of employment may increase the income gap between low-paid and high-paid workers and hence it may not reduce or end social exclusion. Social exclusion may arise from the operations of the market and supplies of key goods and services. For instance, people may not be able to participate in the customary consumption activities because profit maximizing prices may exclude them from the markets. A person may not be allowed to have an account in a bank if he does not fulfil certain constraints. It can as well generate from operations of the State if the State’s social security benefit programmes are targeted towards some particular groups or persons.

As social exclusion includes economic, social and political aspects of life, it is a multidimensional phenomenon. It implies deprivation in a wide range of indicators or functionings of living standards, which can be of quantitative or qualitative type.¹

¹See Atkinson, Cantillon, Marlier and Nolan (2002) for a list of functionings of social

Social exclusion is related to both inequality and poverty, but should not be equated with either of them (Atkinson, 1998). According to Sen (1998), social exclusion is wider than poverty. Multidimensional inequality is a measure of the dispersion of the multidimensional distribution of quantities of consumption of the functionings for different individuals (Tsui, 1999). Multidimensional poverty measurement specifies a poverty threshold for each functioning, looks at the shortfalls of the functioning quantities of different individuals from the threshold levels, and aggregates these shortfalls into an overall magnitude of poverty (Bourguignon and Chakravarty, 2002). Thus, in both cases the functionings have to be of quantitative types, whereas social exclusion considers qualitative type functionings as well. Social exclusion can be regarded as a state and as a process leading to deprivation.

Atkinson (1998) argued further that it is a relative concept, we cannot say whether a person is socially excluded or not by looking at his position alone. The positions of the others in the society have to be taken into account for a proper implementation of any criterion for exclusion. It has, furthermore, a dynamic character because an individual is socially excluded if his deprivation continues or worsens over time.

Three types of implicit conceptualization of social exclusion are currently available in the literature. In the first, it is interpreted as the lack of participation in social institutions (Duffy, 1995, Rowntree Foundation, 1998, UK House of Commons, 1999, Paugam and Russell, 2000); whereas the second regards the problem as the denial or non-realization of rights of citizenship (Room, 1995, Klasen 1998). Finally, the third views social exclusion in terms of increase in distance among population groups (Akerlof, 1997). Some researchers attempted to suggest measures of social exclusion building on these approaches (see Bradshaw et al. 2000, Tsakloglou and Papadopoulos, 2001). However the theoretical foundations of these measures are often unclear.

In this paper we adopt an axiomatic approach to the measurement of social exclusion. Since in order to be socially integrated a person needs to have access to some social functionings, we first look at the functioning failure, that is, the number of functionings from which the person is excluded. We refer to this as the deprivation score or the number of exclusions of the person concerned.

The social exclusion measure that we propose is a real valued function of the deprivation scores of different individuals in the society. In a sense our approach is similar to the view that considers social exclusion as lack

exclusion.

of participation in social institutions, where lack of participation is treated in terms of functioning failures. We characterize the family of exclusion measures whose members satisfy *normalization*, *monotonicity*, *subgroup decomposability*, and have *nondecreasing marginals*. Normalization means that social exclusion is zero if nobody is socially excluded. Monotonicity requires the measure to increase if the deprivation score of a person increases.

According to subgroup decomposability, for any partitioning of the population with respect to some socioeconomic or demographic characteristic, the overall social exclusion is the population share weighted average of subgroup exclusion levels. This property enables us to calculate a particular subgroup's contribution to aggregate exclusion and hence to identify the subgroups that are more afflicted by exclusion and to implement anti-exclusion policy. Clearly, according to this notion of policy recommendation, an assessment of overall exclusion becomes contingent on the implicit valuation of the exclusion measure. However, an exercise of this type may be useful for two reasons. First, following Sen (1985), the non-welfarist approach to policy analysis is becoming quite popular. Second, in many situations policy is evaluated using specific forms of measures. So it seems worthwhile to see what type of policy would be implied by the use of a specific exclusion measure.

Marginal social exclusion is defined as the change in social exclusion when we increase the deprivation score of a person by one. Nondecreasingness of marginal social exclusion ensures that in aggregating individual deprivation scores into an overall indicator of exclusion, a higher deprivation score does not get a lower weight than a lower score.

The characterized family of measures is shown to possess some additional interesting properties. It is also shown that the properties employed in the characterization exercise are independent, that is, none of these properties implies or is implied by another.

Next, we consider the problem of ranking two societies by the social exclusion dominance criterion. We demonstrate that for two societies with a common population size and the same total deprivation score, if one dominates the other by the exclusion dominance criterion, then the former becomes at least as socially excluded as the latter by all additive social exclusion measures that satisfy anonymity and have nondecreasing marginals. This result parallels the if part of the well-known Atkinson (1970) result on Lorenz Domination which says that if u and v are two income distributions of a given total over a fixed population size, and if u Lorenz dominates v , then all symmetric utilitarian social welfare functions regard u at least as good as v , where the identical individual utility function is concave.

Finally, we apply the suggested family of measures to the EU member states in the early 1990's and consider some policy implications.

The paper is organized as follows. The next section introduces the formal framework for measuring social exclusion and presents the properties for an exclusion measure. In Section 3 we characterize the family of exclusion measures and discuss its properties. Section 4 deals with social exclusion dominance relation. The application is contained in Section 5. Section 6 concludes.

2 Properties for a measure of social exclusion

Let \mathbf{N} (\mathbf{N}_0) be the set of all positive (nonnegative) integers and \mathbf{R} be the set of real numbers. For all $n \in \mathbf{N}$, D^n is the n -fold Cartesian product of \mathbf{N}_0 and 1^n is the n -coordinated vector of ones. For any society with a population of size $n \in \mathbf{N}$, there is a finite nonempty set of functionings F relevant for social integration. Throughout this paper we assume that F is fixed so that cross population comparisons of social exclusion can be made in terms of elements of F (see Atkinson et al., 2002, for commonness of elements of F for the EU as a whole). An individual in an n -person society can be excluded from any subset of F , where $n \in \mathbf{N}$ is arbitrary. The degree of exclusion or deprivation of a person can be captured by the number of functionings from which she/he is excluded. Since some functionings may be more important than others, this appears to be a restrictive assumption. We make this assumption for simplicity of exposition. However, our analysis can be extended to allow the possibility that different functionings are given different weights.

An exclusion profile in a society of n person is a vector $x = (x_1, \dots, x_n)$, where $x_i \in \mathbf{N}_0$ is the deprivation score or the number of functionings from which person i is excluded. We assume that the calculation of x_i involves a dynamic or longitudinal aspect.² If x_i is positive, trade-off between excluded and non-excluded functionings is not allowed. For instance, a person's high income cannot compensate the dissatisfaction associated with his job. The set of exclusion profiles for an n -person population is D^n , $n \geq 1$. Thus, $x \in D^n$ for some $n \in \mathbf{N}$. The set of all possible exclusion profiles is $D = \bigcup_{n \in \mathbf{N}} D^n$. A measure of social exclusion is a function $E : D \rightarrow \mathbf{R}$. For any $n \in \mathbf{N}$, the restriction of E on D^n is given by E^n . For any $n \in \mathbf{N}$, $x \in D^n$, $E^n(x)$ is a measure of the extent to which different individuals are excluded from the activities taking place in the society, that is, the degree

²See Section 5 for details.

of exclusion suffered by all individuals in the society as a whole. For all $n \in \mathbf{N}$, $x \in D^n$, let $S(x)$ be the set of persons with positive deprivation scores, that is $S(x) = \{i, 1 \leq i \leq n | x_i > 0\}$. For any $n \in \mathbf{N}$, $x \in D^n$, let q be the cardinality of $S(x)$, that is the number of persons in $S(x)$. For any $n \in \mathbf{N}$, $x \in D^n$, we write \bar{x} for nonincreasingly ordered permutation of x , that is $\bar{x}_1 \geq \bar{x}_2 \geq \dots \geq \bar{x}_n$.

We assume that an arbitrary exclusion measure $E : D \rightarrow \mathbf{R}$ should satisfy the following postulates.

Axiom 1 : *Normalization (NOM)*: For all $n \in \mathbf{N}$, $E^n(01^n) = 0$

Axiom 2 : *Monotonicity (MON)*: For any $n \in \mathbf{N}$, $x \in D^n$ and for any i , $1 \leq i \leq n$,

$$E^n(x) < E^n(x_1, \dots, x_{i-1}, x_i + c, x_{i+1}, \dots, x_n),$$

where $c \in \mathbf{N}$.

Axiom 3 : *Nondecreasingness of Marginal Social Exclusion (NMS)*: For any $n \in \mathbf{N}$, $x \in D^n$, and for any i, j , $1 \leq i, j \leq n$, if $x_i \geq x_j$ then:

$$\begin{aligned} E^n(x_1, \dots, x_{i-1}, x_i + 1, x_{i+1}, \dots, x_{j-1}, x_j, x_{j+1}, \dots, x_n) - E^n(x) &\geq \\ E^n(x_1, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_{j-1}, x_j + 1, x_{j+1}, \dots, x_n) - E^n(x). \end{aligned}$$

Axiom 4 : *Subgroup Decomposability (SUD)*: For $x^i \in D^{n_i}$, $i = 1, \dots, k$, $E^n(x) = \sum_{i=1}^k \frac{n_i}{n} E^{n_i}(x^i)$, where $x = (x^1, x^2, \dots, x^k)$.

Axiom 5 : *Population Principle (POP)*: For all $n \in \mathbf{N}$, $x \in D^n$, $E^{mn}(y) = E^n(x)$, where $y = (x^1, x^2, \dots, x^m)$, $m \geq 1$ and each $x^i = x$.

Axiom 6 : *Anonymity (ANY)*: For all $n \in \mathbf{N}$, $x \in D^n$, $E^n(x) = E^n(xP)$, where P is any $n \times n$ permutation matrix.³

Normalization is a minimality principle. It says that if nobody is excluded from any functioning in the society, then the value of the social exclusion measure is zero. Monotonicity says that if the deprivation score of an individual increases, then social exclusion should increase. If a social exclusion measure satisfies *NOM* and *MON*, then it will take a positive value if at least one individual has a positive deprivation score.

Sen (1976) argued that in income poverty measurement the poverty line can be taken as the reference point for all poor persons and the poverty gap of

³An $n \times n$ matrix is a permutation matrix if each of its entries is either zero or one, and each of its rows and columns sums to one.

a poor, his income shortfall from the poverty line, is a measure of deprivation suffered by him. In order to attach higher weight to higher deprivation, Sen assumed that the weight on individual i 's poverty gap is equal to his rank in the income distribution of the poor. This guarantees that an increase in poverty due to a reduction in the income (increase in deprivation) of a poor will be higher the lower (higher) is the income (deprivation) of the poor. Conversely, in order that an increase in poverty due to reduction in the income of a poor is higher the lower the income of the poor is, a necessary condition is to attach higher weight lower down the income scale. Our *NMS* postulate has a similar spirit. We consider two persons where the deprivation score of the first is not lower than that of the second. Then the change in social exclusion, if the deprivation score of the former increases by one, is at least as large as the corresponding change when the deprivation score of the latter increases by the same amount. It is clear that the postulates *NOM* and *NMS* capture the idea that exclusion is a relative phenomenon.

SUD, which expresses aggregate exclusion in a society as a weighted average of subgroup exclusion levels, where the weights are population shares of the subgroups, is very important from policy point of view. $\frac{n_i}{n} E^{n_i}(x^i)$ is the contribution of subgroup i to total exclusion, i.e., the amount by which social exclusion will decrease if exclusion in subgroup i is eliminated. $\left(\frac{n_i E^{n_i}(x^i)}{n E^n(x)}\right) 100$ is the percentage contribution of subgroup i to total exclusion. Each of these figures is useful to planners and analysts to formulate anti-exclusion policies.

POP says that if an exclusion profile is replicated several times, then the social exclusion of the original and the replicated profiles are the same. Clearly, *POP* leads us to view exclusion in average terms. It is helpful for cross population comparisons of exclusions.

Finally, *ANY* means that the exclusion measure is symmetric, i.e. any reordering of the deprivation scores leaves the exclusion level unchanged. *ANY* is unavoidable as long as the individuals are not distinguished by anything other than deprivation scores.

3 The family of subgroup decomposable social exclusion measures

In this section we derive the class of social exclusion measures whose members satisfy *NOM*, *MON*, *NMS*, in addition to *SUD*. Let Φ be the class of all functions $f : \mathbf{N}_0 \rightarrow \mathbf{R}$ such that $f(0) = 0$, f is increasing, and f has a

nondecreasing marginal, that is:

$$f(x_i + 1) - f(x_i) \geq f(x_j + 1) - f(x_j), \quad (1)$$

where $x_i \geq x_j$.

We then have:

Theorem 1 : A social exclusion measure $E : D \rightarrow \mathbf{R}$ satisfies *NOM*, *MON*, *NMS*, and *SUD* if and only if for all $n \in \mathbf{N}$, $x \in D^n$,

$$E^n(x) = \frac{1}{n} \sum_{i \in S(x)} f(x_i), \quad (2)$$

where f is a member of Φ .

Proof:

Let $n \in \mathbf{N}$ and $x \in D^n$ be arbitrary. Then by repeated applications of *SUD*:

$$E^n(x) = \frac{1}{n} \sum_{i=1}^n E^1(x_i). \quad (3)$$

We can rewrite E^n in (3) as:

$$E^n(x) = \frac{1}{n} \sum_{i=1}^n f(x_i), \quad (4)$$

where $f = E^1$. Clearly, $f : \mathbf{N}_0 \rightarrow \mathbf{R}$. *MON* demands increasingness of f . Now, suppose $x_i \geq x_j$. The inequality:

$$\begin{aligned} E^n(x_1, \dots, x_{i-1}, x_i + 1, x_{i+1}, \dots, x_{j-1}, x_j, x_{j+1}, \dots, x_n) - E^n(x) &\geq \\ E^n(x_1, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_{j-1}, x_j + 1, x_{j+1}, \dots, x_n) - E^n(x), \end{aligned}$$

on simplification, reduces to:

$$\begin{aligned} f(x_i + 1) - f(x_i) &\geq \\ f(x_j + 1) - f(x_j), \end{aligned}$$

which is nondecreasingness of marginal of f . Clearly, if $x_i = 0$ for all i , then *NOM* requires that $f(0) = 0$.

Obviously, $f(0) = 0$ enables us to rewrite $\frac{1}{n} \sum_{i=1}^n f(x_i)$ as $\frac{1}{n} \sum_{i \in S(x)} f(x_i)$. This establishes the necessity part of the theorem on D^n for a given $n \in \mathbf{N}$.

The sufficiency is easy to verify. Since $n \in \mathbf{N}$ was chosen arbitrarily, our result holds for all $n \in \mathbf{N}$. ■

Note that the general measure in (2) satisfies *ANY* and *POP* although we did not use these properties in its derivation. We can interpret f in (2) as the individual exclusion function. An alternative way of writing the formula (2) is:

$$E^n(x) = \frac{H}{q} \sum_{i \in S(x)} f(x_i), \quad (5)$$

where $H = \frac{q}{n}$ is the head-count measure of social exclusion, the proportion of persons that is socially excluded in the population. For a fixed n , on social exclusion profiles with a given q , H is a constant function. Thus H is violator of *MON* although it meets *NOM*, *SUD*, *POP*, *ANY*, and *NMS*.

In order to illustrate the general formula in (2), let $f \in \Phi$ be of the form $f(t) = t^\delta$, $\delta \geq 1$. Then the corresponding measure is:

$$E_\delta^n(x) = \frac{H}{q} \sum_{i \in S(x)} x_i^\delta. \quad (6)$$

For any $\delta \geq 1$, E_δ^n satisfies all the postulates. For $0 < \delta < 1$, E_δ^n is a violator of *NMS* but not of others. As $\delta \rightarrow 0$, $E_\delta^n \rightarrow H$. The single parameter δ in (6) is a value judgement parameter. E_δ^n becomes more sensitive to the higher deprivation scores as δ increases from 1 to plus infinity. For a given $x \in D^n$, an increase in the value of δ does not decrease E_δ^n . For $\delta = 1$, E_δ^n becomes the average deprivation score of the society, that is, $A = \frac{1}{n} \sum_{i \in S(x)} x_i$. For $\delta = 2$, we can rewrite E_δ^n as:

$$E_\delta^n(x) = H \left[\sigma^2(x) + (A(x))^2 \right], \quad (7)$$

where σ^2 is the variance of the deprivation scores. Given H and A , a reduction in σ^2 reduces the measure in (7). Such a situation may arise if a higher deprivation score decreases and a lower deprivation score increases by the same amount. Over social exclusion profiles with the same head-count measure and the same average deprivation score, the ranking of the profiles generated by E_δ^n (for $\delta = 2$) is the same as that generated by σ^2 .

An alternative of interest arises from the specification $f(t) = e^{\alpha t} - 1$, where $\alpha > 0$. The resulting measure is:

$$E_\alpha^n(x) = \frac{H}{q} \sum_{i \in S(x)} (e^{\alpha x_i} - 1). \quad (8)$$

For a given $x \in D^n$, E_α^n is nondecreasing in α . E_α^n satisfies all the properties for all positive α . As α increases, the underlying evaluation attaches more weight to the higher deprivation scores.

We will now show that the postulates *NOM*, *MON*, *NMS* and *SUD* are independent. By independence we mean that if one of these postulates is dropped, then there will be measures that will satisfy the remaining postulates, but not the dropped one.

Theorem 2 : *The properties NOM, MON, NMS, and SUD are independent.*

Proof:

(a) Evidently the measure $I_1^n(x) = \frac{1}{n} \sum_{i \in S(x)} e^{x_i}$ is not normalized, but it will fulfil the other properties.

(b) Since the measure $I_2^n(x) = -\frac{1}{n} \sum_{i \in S(x)} \frac{x_i}{1+x_i}$ is decreasing in x_i , it is a violator of *MON*, but not of the remaining postulates.

(c) The measure $I_3^n(x) = \frac{1}{n} \sum_{i \in S(x)} x_i^\theta$, $0 < \theta < 1$, has a decreasing marginal and hence it fails to satisfy *NMS*, but it verifies the other properties.

(d) Since the measure $I_4^n(x) = \left(\frac{1}{n} \sum_{i \in S(x)} x_i^\nu \right)^{\frac{1}{\nu}}$, $\nu > 1$, is not additive across components, it is not subgroup decomposable. However, it is normalized, monotonic, and has increasing marginals. ■

It is clear that to every individual exclusion function $f \in \Phi$, there corresponds a different social exclusion measure of the form (2). They will differ only in the manner how a person's individual exclusion is specified as a function of his deprivation score. However, there is no guarantee that these social exclusion measures will rank exclusion profiles in the same way. We consider the problem of ranking exclusion profiles in the next section.

4 The social exclusion dominance relation

We begin this section by defining the social exclusion dominance criterion and look at its implications for exclusion profiles with a fixed total over a given population size.

For $x, y \in D^n$, we say that x dominates y by the social exclusion relation, which we write $x \succeq_{SE} y$, if:

$$\sum_{j=1}^k \bar{x}_j \geq \sum_{j=1}^k \bar{y}_j, \quad (9)$$

for all $k = 1, 2, \dots, n$.

Given that the exclusion profiles \bar{x} and \bar{y} are ranked in nonincreasing order of functioning failures of the individuals, $x \succeq_{SE} y$ demands that the cumulative deprivation score of the first k persons in \bar{x} is at least as large as that in \bar{y} , where $k = 1, 2, \dots, n$.

In order to study implications of the relation \succeq_{SE} in terms of exclusion measures, we now have the following:

Definition 1 : For any $x \in D^n$, we say that \bar{y} is obtained from \bar{x} by a favourable composite change (FCC) if:

$$\begin{aligned} \bar{y}_i &= \bar{x}_i - 1 \\ \bar{y}_j &= \bar{x}_j + 1 \\ \bar{y}_k &= \bar{x}_k \quad \text{for all } k \neq i, j, \end{aligned} \tag{10}$$

where $\bar{x}_i > \bar{x}_j$.

An FCC, which explicitly involves relativity issue of exclusion, means that the degree of exclusion of a more deprived person (i) is reduced by 1, whereas that of a less deprived person (j) is increased by 1, so that the total scores in the two profiles are the same, but the variance of the new profile (\bar{y}) is less than that of the original one (\bar{x}). Note that the transformation in (10) does not alter the relative positions of the affected individuals and it reduces the functioning failure of the worse off person (i). This is the reason why we call it an FCC.

Marshall and Olkin (1979) defined a special kind of linear transformation, called a T -transformation, of a vector that leaves all but two components of the vector unchanged, and replaces these two components by averages. An FCC is a T -transformation since:

$$\begin{aligned} \bar{y}_i &= \lambda \bar{x}_i + (1 - \lambda) \bar{x}_j \\ \bar{y}_j &= (1 - \lambda) \bar{x}_i + \lambda \bar{x}_j \\ \bar{y}_k &= \bar{x}_k \quad \text{for all } k \neq i, j, \end{aligned} \tag{11}$$

where $\lambda = \frac{(\bar{x}_i - \bar{x}_j - 1)}{(\bar{x}_i - \bar{x}_j)}$.

The following theorem gives an interesting consequence of the relation \succeq_{SE} for additive exclusion measures that satisfy anonymity and have non-decreasing marginals.

Theorem 3 : Let $x, y \in D^n$, where $\sum_{l=1}^n x_l = \sum_{l=1}^n y_l$. Then $x \succeq_{SE} y$ implies that $\sum_{l=1}^n h(x_l) \geq \sum_{l=1}^n h(y_l)$ for all individual exclusion measures $h : \mathbf{N}_0 \rightarrow \mathbf{R}$ whose marginals are nondecreasing.

Proof:

Muirhead (1903) showed that given $x, y \in D^n$ along with $\sum_{l=1}^n x_l = \sum_{l=1}^n y_l$, if $x \succeq_{SE} y$ holds, then \bar{y} can be derived from \bar{x} by successive applications of a finite number of *FCCs*. Assume, without loss of generality, that only one *FCC* affecting individuals i and j , where $\bar{x}_i > \bar{x}_j$, takes us from \bar{x} to \bar{y} .

Given $\bar{x}_i > \bar{x}_j$, let $\theta = \bar{x}_i - \bar{x}_j - 1$. Note that $\theta \in \mathbf{N}_0$. Since the marginal of the individual exclusion function h is nondecreasing, we have:

$$h(\bar{x}_j + 1) - h(\bar{x}_j) \leq h(\bar{x}_j + \theta + 1) - h(\bar{x}_j + \theta), \quad (12)$$

which we can rewrite as:

$$h(\bar{x}_j + 1) - h(\bar{x}_j) \leq h(\bar{x}_i) - h(\bar{x}_i - 1). \quad (13)$$

Inequality (13) on rearrangement gives:

$$h(\bar{x}_j + 1) + h(\bar{x}_i - 1) \leq h(\bar{x}_i) + h(\bar{x}_j). \quad (14)$$

Substituting the values of $\bar{x}_j + 1$ and $\bar{x}_i - 1$ in (14), we get:

$$h(\bar{y}_j) + h(\bar{y}_i) \leq h(\bar{x}_i) + h(\bar{x}_j). \quad (15)$$

Inequality (15) along with the information that $\bar{y}_k = \bar{x}_k$ for all $k \neq i, j$ gives us:

$$\sum_{l=1}^n h(\bar{y}_l) \leq \sum_{l=1}^n h(\bar{x}_l). \quad (16)$$

Since the social exclusion measure $\sum h(\cdot)$ satisfies anonymity, we can rewrite (16) as:

$$\sum_{l=1}^n h(y_l) \leq \sum_{l=1}^n h(x_l), \quad (17)$$

which is the desired result. ■

Theorem 3 is very valuable. It shows how an *FCC* becomes helpful in ranking two exclusion profiles. It also provides a justification for using *NMS* as a postulate for a social exclusion measure.

In an *FCC* the deprivation scores of the two affected persons change in opposite directions. But often unidirectional changes in the scores of the two or more persons may take place. The following result, whose proof can be found in Fulkerson and Ryser (1962), states that under certain conditions the relation $x \succeq_{SE} y$, where the total scores in x and y are the same, is preserved.

Theorem 4 : Let $x, y \in D^n$, where $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$, be arbitrary. Then $x \succeq_{SE} y$ implies that $(\bar{x} - e_j) \succeq_{SE} (\bar{y} - e_i)$, where $i \leq j$ and e_k is the n -coordinated vector with 1 in the k^{th} position and zeros elsewhere.

The intuitive appeal of Theorem 4 is quite clear. Given that x dominates y if we reduce the degree of exclusion of one person in \bar{x} and one person in \bar{y} , where the latter is relatively worse off than the former, the exclusion dominance remains preserved.

The following result, whose formal proof can be found in Fulkerson and Ryser (1962), is a generalization of Theorem 4.

Theorem 5 : Let $x, y \in D^n$, where $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$, be arbitrary. Let u be obtained from \bar{x} by reducing deprivation scores of persons in positions i_1, i_2, \dots, i_k by 1. Similarly, suppose v is obtained from \bar{y} by reducing deprivation scores of persons in positions j_1, j_2, \dots, j_k . If $i_1 \leq j_1, i_2 \leq j_2, \dots, i_k \leq j_k$ and $x \succeq_{SE} y$, then $u \succeq_{SE} v$.

5 An empirical illustration

The purpose of this section is to illustrate the social exclusion measure E_δ in (6) using the European Community Household Panel (ECHP) data.⁴ We base our analysis on the first five waves of ECHP, which cover the period from 1993 to 1997. The surveys are conducted at a European national level. The ECHP is an ambitious effort at collecting information on the living standards of the households of the EU member-states using common definitions, information collection methods and editing procedures. It contains detailed information on incomes, socio-economic characteristics, housing amenities, consumer durables, social relations, employment conditions, health status, subjective evaluation of well-being, etc. Of the 15 EU member-states, we could not consider Austria, Finland, Luxembourg, and Sweden since the data for these countries were not available for all the waves. For similar reasons we had to exclude Germany and the UK. In particular, the ECHP surveys of these countries were substituted by national surveys, SOEP and BHPS respectively, that did not collect information on all the variables considered in our application.

Information have been collected at the individual or the household level depending on the variable, but the unit of our analysis is the individual. The

⁴Since our illustration involves cross population comparisons, we drop the superscript n from E_δ^n .

calculation uses required sample weights. In ECHP a person's life has been measured along the following domains: financial difficulties, basic needs and consumption, housing conditions, durables, health, social contacts and participation, and life satisfaction. The 14 non-monetary indicators⁵ suggested by Eurostat (2000) as best candidates to meet the requirements of 1) reflecting a negative aspect of a life pattern common to a majority of the population in the EU; 2) allowing international and intertemporal comparisons; 3) expressing a link with income poverty, are included in the analysis. These are the following:

- Financial difficulties: 1. Proportion of persons living in households that have great difficulties in making ends meet; 2. Proportion of persons living in households that are in arrears with (re)payment of housing and/or utility bills;
- Basic necessities: 3. Proportion of persons living in households which cannot afford meat, fish or chicken every second day; 4. Proportion of persons living in households which cannot afford to buy new clothes; 5. Proportion of persons living in households which cannot afford a week's holiday away from home;
- Housing conditions: 6. Proportion of persons living in the accommodation without a bath or shower; 7. Proportion of persons living in the dwelling with damp walls, floors, foundations, etc.; 8. Proportion of persons living in households which have a shortage of space;
- Durables: 9. Proportion of persons not having access to a car due to a lack of financial resources in the household; 10. Proportion of persons not having access to a telephone due to a lack of financial resources in the household; 11. Proportion of persons not having access to a color TV due to a lack of financial resources in the household;
- Health: 12. Proportion of persons (over 16) reporting bad or very bad health;
- Social contact: 13. Proportion of persons (over 16) who meet their friends or relatives less often than once a month (or never);

⁵In fact, the non-monetary indicators recommended in Eurostat (2000) are 15. We decided to drop the one belonging to the health domain, namely the proportion of people that were severely hampered in their daily activity by long-lasting health problems, since there was a considerable discontinuity between the ECHP waves for this indicator.

- Dissatisfaction: 14. Proportion of persons (over 16) being dissatisfied with their work or main activity.

We calculate E_δ for $\delta = 0, 1,$ and 2 separately for two sets of indicators V_1 and V_2 , where V_1 includes the indicators in the domains of financial difficulties, basic necessities, housing conditions, and durables, and V_2 includes the remaining indicators. The reason for separate calculations is that for indicators covered under V_1 we have household level information, whereas for the indicators in V_2 the available information are at the individual level, with the additional constraint that the minimum age of the reportee is 16.

We call a person socially excluded with respect to a variable in a given domain if he has been deprived of the variable for at least three years, two of which have to be consecutive in the last four years. A person's exclusion in a given domain has been obtained by adding up his exclusions over the concerned variables. Thus, our calculation of the individual exclusion score explicitly takes into account the dynamic or longitudinal aspect of social exclusion.

Numerical estimates of social exclusion for the EU member states are reported in Table 1. The upper part of the table presents the estimates for V_1 while its lower part gives the analogous values for V_2 . The first column of the table gives the names of the countries for whom required information were available. In columns 2-4 we present, for each country, the values of E_δ for $\delta = 0, 1$ and 2 respectively.⁶ The country-wise social exclusion levels are then weighted by corresponding population shares to determine the contributions of different countries to total exclusion, which are given as percentages of total exclusion in columns 5-7. From policy perspective, complete elimination of exclusion within a country would lower aggregate exclusion precisely by the percentage by which it contributes to total exclusion.

Several interesting features emerge from Table 1. We note that the values of measures as well as percentage contributions are sensitive to the values of δ . Increasingness of E_δ as δ increases is confirmed by values of measures shown in the table. We first analyze the upper part of the table. Portugal turns out to be the most socially excluded country. On the other hand, the country with the least social exclusion according to H is the Netherlands, whereas E_1 and E_2 regard Denmark as the country with minimal exclusion. Similarly, for the remaining seven countries, ranking by one measure differs from that generated by another.

⁶Recall that for $\delta = 0$ and 1 , E_δ becomes respectively the head-count ratio, H , and the average deprivation score of the society, A .

As expected, the maximum percentage contribution to total exclusion comes from Portugal, whereas Denmark is the least contributing country. Belgium occupies the second position in terms of low percentage contribution. The fourth column of this part of the table shows that Portugal, Italy, Spain, and Greece, the Southern European countries, report 77.62% of social exclusion as judged by the headcount index. Their contribution to overall exclusion rises to 81.64% (85.66%) if one uses A (E_2). The higher contributions of these four countries is partly due to their almost average or more than average social exclusions. France and Ireland come next in the ranking by percentage contributions by H and A . A comparison between Italy and Ireland is worth noting here. Although the latter has a better position than the former with respect to H , for the two other measures it becomes worse off. The reason behind is that the average exclusion of the excluded person for $\delta = 1$ and 2, that is, $\frac{A}{H}$ and $\frac{E_2}{H}$ are much higher in Ireland than in Italy. By percentage contributions, Ireland shows a much better picture than Italy. This is because the country has a very low population share among the member states.

In V_2 as well, Portugal is the member state with maximum exclusion and percentage contribution, and Italy has the second worst off position with respect to both the factors. If we consider ranking of the countries from low to high exclusion and percentage contributions, then for five countries the sequence is Denmark, Ireland, the Netherlands, Belgium and Greece. The ranking between France and Spain is ambiguous by either of the two criteria. Portugal, Italy, Spain and France jointly contribute more than 83% to total exclusion by any measure. Finally, it is worth noting that, except for Portugal, the ranking of countries by any measure in V_2 is different from that in V_1 .

From policy point of view, the break-down of the variables into two subgroups enable us to identify the countries separately in each subset that are most susceptible to exclusion.

In Table 2 we carry out a similar analysis for Italy. The country has been divided into 11 geographic areas.⁷ In V_1 , where there is no unanimous agreement about the area with maximum exclusion, Emilia-Romagna has emerged as the region with minimum exclusion. In contrast, in V_2 , South is the geographic area with maximum exclusion and Lombardia has the lowest exclusion. More generally, ranking of areas by any measure is different in V_1 than that in V_2 . The subgroup with maximum and minimum contributions

⁷The information on the geographic areas of the Italian households are available in ECHP at the Nuts 1 level.

are the same in both V_1 and V_2 , namely South and Emilia-Romagna. The total contribution of the last four geographic areas is more than 52% in both the situations.

The high contributing areas require attention from policy perspective for reduction of their contributions so that a higher living standard can be achieved.

6 Conclusions

Social exclusion refers to inability of a person to participate in basic day-to-day economic and social activities of life.

In this paper we have developed an axiomatic approach to the measurement of social exclusion and identified the class of subgroup decomposable measures of exclusion. We have also suggested a dominance criterion for ranking two societies by symmetric additive exclusion measures under constancy of population size and total deprivation score. A policy application of our measure have been suggested in the paper using European Union data.

Several extensions of our analysis are possible. First, a characterization of some class of measures, for example of E_δ , will be quite interesting. Second, extension of our dominance criterion to the cases of nonadditive measures, variable total and variable population size will be worthwhile. Finally, we have considered only subgroup decomposability. We can as well consider decomposition of population exclusion by attributes and study the impact of each of them on the aggregate exclusion. This will enable us to identify the attributes that are more/less susceptible to social exclusion.

References

- [1] Akerlof, G.A. (1997): "Social distance and social decision", *Econometrica*, 65, 1005–1027.
- [2] Atkinson, A.B. (1970): "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244-263.
- [3] Atkinson, A.B. (1998): "Social exclusion, poverty and unemployment", CASE/4, Centre for Analysis of Social Exclusion, London School of Economics, 1-20.
- [4] Atkinson, A.B., B. Cantillon, E. Marlier, and B. Nolan (2002): *Social Indicators: The EU and Social Inclusion*, Oxford.

- [5] Bourguignon, F. and S.R. Chakravarty (2002): “The Measurement of Multidimensional Poverty”, *Journal of Economic Inequality*, forthcoming.
- [6] Bradshaw, J., Williams, J., Levitas, R., Pantazis, C., Patsios, D., Townsend, P., Gordon, D. and S. Middleton (2000): “The Relationship Between Poverty and Social Exclusion in Britain”, Paper at the 26th General Conference of the International Association for Research in Income and Wealth, Cracow, Poland.
- [7] Duffy, K. (1995): “Social exclusion and human dignity in Europe”, Council of Europe, Strasbourg.
- [8] Eurostat (2000): “European social statistics. Income, poverty and social exclusion,” THEME 3, Population and social conditions, Luxembourg.
- [9] Fulkerson, D.R. and H.J. Ryser (1962): “Multiplicities and Minimal Widths for (0-1) Matrices”, *Canadian Journal of Mathematics*, 14, 498-508.
- [10] Klasen, S. (1998): “Social exclusion and children in OECD countries: some conceptual issues”, Centre for Educational Research and Innovation, OECD.
- [11] Marshall, A.W. and I. Olkin (1979): *Inequalities: theory of majorization and its applications*, Academic Press.
- [12] Mejer, L. (2000): “Statistics on social exclusion: the EU methodological approach”, Eurostat, Unit E2 Living Conditions.
- [13] Muirhead, R.F. (1903): “Some Methods Applicable to Identities and Inequalities of Symmetric Algebraic Functions of n Letters”, *Proceedings of Edinburgh Mathematical Society*, 21, 144-157.
- [14] Paugam, S. and H. Russell (2000): “The effects of employment precarity and unemployment on social isolation”, in: D. Gallie and S. Paugam (eds.), *Welfare regimes and the experience of unemployment in Europe*, Oxford University Press, Oxford.
- [15] Room, G. (1995): *Beyond the threshold: the measurement and analysis of social exclusion*, Policy Press, Bristol.
- [16] Rowntree Foundation (1998): “The report of key indicators of poverty and social exclusion”, www.parliament.the-stationery-office.co.uk/pa/cm199798/cmselect/777/8060303.htm.

- [17] Sen, A.K. (1976): “Poverty: an ordinal approach to measurement”, *Econometrica*, 44, 219–231.
- [18] Sen, A.K. (1985): *Commodities and Capabilities*, North Holland, Amsterdam.
- [19] Sen, A.K. (1998): “Social exclusion and economic measurement”, paper presented at the 25th General Conference of the International Association for Research in Income and Wealth, Cambridge, UK.
- [20] Tsakloglou, P. and F. Papadopoulos (2001): “Identifying population groups at high risk of social exclusion”, forthcoming in: R. Muffels and P. Tsaklogou (eds.), *Social exclusion in European welfare states*, Edward Elgar, Cheltenham.
- [21] Tsui, K-Y. (1999): “Multidimensional inequality and multidimensional generalized entropy measures: An axiomatic derivation”, *Social Choice and Welfare*, 16, 145–157.
- [22] UK House of Commons (1999): “Poverty and social exclusion (national strategy) bill”, www.parliament.the-stationery-office.co.uk/pa/cm199899/cmbills/045/1999045.htm.

Table 1: Social Exclusion in EU Member-States (1993-1997).

	Values of E_s			Percentage Contributions Based on:		
	E_0 (head-count ratio, H)	E_1 (average deprivation score, A)	E_2	E_0 (head-count ratio, H)	E_1 (average deprivation score, A)	E_2
V1						
Belgium	0.248	0.414	1.066	2.84	2.14	1.63
Denmark	0.212	0.302	0.570	2.02	1.30	0.72
Greece	0.680	1.861	7.233	18.08	22.28	25.60
Spain	0.573	1.060	2.748	19.28	16.06	12.30
France	0.377	0.622	1.489	9.29	6.90	4.88
Ireland	0.427	0.910	3.013	5.23	5.02	4.92
Italy	0.496	0.871	2.24	17.88	14.14	10.75
Netherlands	0.191	0.318	0.778	4.01	3.00	2.18
Portugal	0.752	2.278	9.779	21.38	29.16	37.01
Total	0.548	1.216	4.114	100	100	100
V2						
Belgium	0.075	0.079	0.085	2.98	2.78	2.45
Denmark	0.001	0.002	0.003	0.04	0.06	0.09
Greece	0.090	0.095	0.106	9.40	8.87	8.04
Spain	0.110	0.113	0.118	14.14	12.88	10.88
France	0.095	0.108	0.136	10.90	11.02	11.25
Ireland	0.027	0.029	0.033	1.30	1.25	1.16
Italy	0.186	0.217	0.286	28.05	29.15	31.12
Netherlands	0.035	0.038	0.044	3.04	2.92	2.72
Portugal	0.268	0.310	0.398	30.15	31.08	32.29
Total	0.136	0.152	0.188	100	100	100

V1 considers jointly the variables included in the domains of financial difficulties, basic necessities, housing conditions, durables. V2 considers jointly the variables included in the domains of health, social contact and dissatisfaction. The values reported are per persons, with the additional constraint of age being at least 16 for V2.

Table 2: Social Exclusion in Italy by Geographic Areas (1993-1997).

	Values of E_s			Percentage Contributions Based on:		
	E_0 (head-count ratio, H)	E_1 (average deprivation score, A)	E_2	E_0 (head-count ratio, H)	E_1 (average deprivation score, A)	E_2
V1						
North West	0.250	0.371	0.734	4.71	3.99	3.07
Lombardia	0.193	0.245	0.396	4.02	2.91	1.82
North East	0.238	0.342	0.611	5.84	4.79	3.32
Emilia-Romagna	0.249	0.295	0.437	2.98	2.01	1.16
Centre	0.430	0.631	1.274	11.38	9.51	7.46
Lazio	0.492	0.789	1.792	6.50	5.94	5.25
Abruzzo-Molise	0.453	0.662	1.257	7.03	5.85	4.32
Campania	0.531	1.021	2.807	11.23	12.29	13.13
South	0.706	1.508	4.440	21.22	25.80	29.54
Sicilia	0.700	1.303	4.234	13.38	14.18	17.92
Sardegna	0.756	1.445	3.800	11.70	12.72	13.01
Total	0.496	0.871	2.240	100	100	100
V2						
North West	0.124	0.148	0.204	6.10	6.22	6.50
Lombardia	0.110	0.129	0.173	6.20	6.21	6.30
North East	0.114	0.134	0.174	7.40	7.45	7.34
Emilia-Romagna	0.125	0.137	0.166	3.89	3.66	3.37
Centre	0.172	0.199	0.259	11.81	11.75	11.57
Lazio	0.171	0.212	0.298	6.12	6.49	6.92
Abruzzo-Molise	0.142	0.160	0.196	5.74	5.52	5.12
Campania	0.231	0.264	0.334	13.42	13.17	12.62
South	0.249	0.296	0.404	19.96	20.37	21.05
Sicilia	0.179	0.211	0.291	9.18	9.31	9.72
Sardegna	0.237	0.268	0.339	10.18	9.85	9.47
Total	0.186	0.217	0.286	100	100	100

V1 considers jointly the variables included in the domains of financial difficulties, basic necessities, housing conditions, durables. V2 considers jointly the variables included in the domains of health, social contact and dissatisfaction. The values reported are per persons, with the additional constraint of age being at least 16 for V2.