Second-order estimates in anisotropic elliptic problems

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In recent years, various results showed that second-order regularity of solutions to the \( p \)-Laplace equation can be properly formulated in terms of the expression under the divergence, the so-called stress field.

I will discuss the extension of these results to the anisotropic \( p \)-Laplace problem, namely equations of the kind

\[
-\text{div} \left( A(\nabla u) \right) = f \quad \text{in} \; \Omega, \tag{1}
\]

in which the stress field is given by \( A(\nabla u) = H^{p-1}(\nabla u) \nabla \xi H(\nabla u) \), for a given norm \( H = H(\xi) \) on \( \mathbb{R}^n \) satisfying suitable ellipticity assumptions.

The \( W^{1,2} \)-Sobolev regularity of \( A(\nabla u) \) is established when \( f \) is square integrable, and both local and global estimates are obtained. The latter apply to solutions to homogeneous Dirichlet problems on either convex or sufficiently regular domains \( \Omega \).

A key point in our proof is an extension of Reilly's identity to the anisotropic setting.

This is based on joint works with A. Cianchi, G. Ciraolo, A. Farina and V.G. Maz'ya [1, 2].

References
