



UNIVERSITÀ DI PARMA

DIPARTIMENTO DI SCIENZE MATEMATICHE, FISICHE E INFORMATICHE

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Seminari di Fisica Matematica

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Giovedì 13 novembre 2025 ore 11:00

Aula F, Plesso di Matematica

A well suited approach of some relaxation operators to moment methods. Derivation and analysis.

Abstract: Kinetic models are used to simulate rarefied gas in the context of atmospheric reentry, CVI deposition, microchannels and other processes. The question arises whether the very fine description of the gas that is given by the original Boltzmann equation is required or not for such simulations. Depending on the Knudsen number, the collision operator may be replaced by simpler models that are easier to handle. And among them we focus on relaxation operators such as the seminal BGK model. Here, we aim to develop a theoretical approach that applies to existing models and serves as a ground from which ongoing models can be constructed. For short, the construction is based on relaxation equations that are relations between some moments of the operator and some moments of the distribution function. Those relations include parameters related to a given vector of tensors that are fitted in order to approach the linearized Boltzmann operator while keeping the structure of the original equation. Denoting the relaxation operator with $R(f) = \nu(G-f)$, they are restated in term of a linear application between the moments of f to those of G . The first mathematical problem that arises is to characterize those linear applications that go from the set moments of nonnegative functions into itself. This implies to have a tractable way to describe this set. The next problem which is also at the center of moment methods is the way to define G ones its moments are known. For most relaxation operators, it is usually done by minimizing the natural entropy under moment constraints. Unfortunately, this functional is not suited neither to the simple case of Grad thirteen



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moments nor in general to higher degree. Junk was the first to raise and characterize the problem that one may face with Levermore's closure. He was then followed by different authors giving their own insight on the problem and have eventually proposed some way to fix it. Our purpose is to come back to some general conditions for such problems as settled by Borwein and Lewis (1991) and then by Csiszar (1995), point out the problem for the classical entropy functional and then derive a "simple" theorem with reasonable assumptions on the functional to be minimized in order to obtain a unique solution qualifying all constraints. This solution also in an analytical form (up the computation of the dual variables). We also want to mention M. Abdelmalik and H. Van Brummelen article (2016) making use of Csiszar results and partly motivating us in the present study. As a consequence, one can prove that the model which is constructed just basing on relaxations on the Grad 13 moments is well posed. It also almost satisfies the decay of the Boltzmann entropy in the polynomial approximation of the exponential function. We finish our talk by displaying some known models that can be derived in this framework just by changing the functional to be minimized and giving some perspectives.

Organizzatori: Andrea Bondesan, Maria Groppi



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