The problem of infinite propagation speed in diffusion equations

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One of the most widely used mathematical tools in modeling is the class of diffusion equations, which are present not only in physical, chemical, and biological models, but also in virtually every scientific field. For example, the now famous Black–Scholes equation for European call options is nothing but a diffusion equation in which what diffuses are prices.

Although it is well known that linear diffusion models based on Fick's law (just as the heat equation based on Fourier's law) lead to the physical contradiction of infinite propagation speed, they are still the most commonly used models whenever some diffusion process is involved. In certain specific problems, the models obtained using these linear equations provide a good approximation to reality, despite the aforementioned physical contradiction, because the solutions, although positive, are very small outside a compact set. However, in many biological models, such as morphogen transport, the infinite propagation speed completely invalidates the model.

In this talk, after presenting a historical overview of the different diffusion models, we introduce some of the results we have obtained concerning a diffusion equation proposed by Ph. Rosenau, and independently by Y. Brenier. This equation, referred to by Y. Brenier as the *relativistic heat equation*, has finite propagation speed and, more interestingly, the maximum propagation speed is a parameter of the equation itself. Therefore, it can be predetermined according to the nature of the problem under study.